Reaction : $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$

The rate is calculated with a variant of the model presented in NO85 and LA86c,

$$N_{\mathrm{A}}^2\langle\sigma v\rangle^{\alpha\alpha\alpha} = 3N_{\mathrm{A}}\left(\frac{8\pi\hbar}{\mu_{\alpha\alpha}^2}\right)\left(\frac{\mu_{\alpha\alpha}}{2\pi k_{\mathrm{B}}T}\right)^{3/2}\int_0^\infty \frac{\sigma_{\alpha\alpha}(E)}{\Gamma_\alpha(^8\mathrm{Be},E)}\exp(-E/k_{\mathrm{B}}T)N_{\mathrm{A}}\langle\sigma v\rangle^{\alpha^8\mathrm{Be}}\,E\,dE,$$

where $\mu_{\alpha\alpha}$ is the reduced mass of the α + α system, and E is the energy with respect to the α + α threshold. The elastic cross section of α + α scattering is given by Eq. (7) with $\Gamma_i = \Gamma_f = \Gamma_\alpha(^8\mathrm{Be},E)$. The energy-dependent width of the ^{8}Be ground state is defined in Eq. (9). The energy $E_{^8\mathrm{Be}}$ and width $\Gamma_\alpha(^8\mathrm{Be})$ of the ^{8}Be ground state are displayed in the Table. The $N_{\mathrm{A}}\langle\sigma v\rangle^{\alpha^8\mathrm{Be}}$ rate assumes that ^{8}Be has been formed at an energy E different from $E_{^8\mathrm{Be}}$, and that it is bound [LA86c]. This rate is given by

$$N_{\mathrm{A}}\langle\sigma v\rangle^{\alpha^8\mathrm{Be}} = N_{\mathrm{A}}\frac{8\pi}{\mu_{\alpha^8\mathrm{Be}}^2}\left(\frac{\mu_{\alpha^8\mathrm{Be}}}{2\pi k_{\mathrm{B}}T}\right)^{3/2}\int_0^\infty \sigma_{\alpha^8\mathrm{Be}}(E';E)\exp(-E'/k_{\mathrm{B}}T)\ E'\,dE',$$

where $\mu_{\alpha^8\mathrm{Be}}$ is the reduced mass of the α + ^{8}Be system, and E' is the energy with respect to its threshold (which varies with the formation energy E). As in NO85 and LA86c, we parametrize $\sigma_{\alpha^8\mathrm{Be}}(E';E)$ as

$$\sigma_{\alpha^8\mathrm{Be}}(E';E) = \sum_{J=0,2}(2J+1)\ \frac{\pi\hbar^2}{2\mu_{\alpha^8\mathrm{Be}}E'}\ \frac{\Gamma_\alpha(^{12}\mathrm{C}^J,E')\Gamma_\gamma(^{12}\mathrm{C}^J,E'+E)}{(E'-E_{\mathrm{r}}^J+E-E_{^8\mathrm{Be}})^2+\frac{1}{4}\Gamma(^{12}\mathrm{C}^J,E';E)^2},$$

where the sum runs over the 0_2^+ resonance and an assumed 2^+ resonance at energies E_{r}^J. The widths are given by $\Gamma = \Gamma_\alpha+\Gamma_\gamma$, $\Gamma_\alpha(^{12}\mathrm{C}^J,E') = \Gamma_\alpha(^{12}\mathrm{C}^J)\ P_l(E')/P_l(E_{\mathrm{r}}^J)$ and $\Gamma_\gamma(^{12}\mathrm{C}^J,E'+E) = \Gamma_\gamma(^{12}\mathrm{C}^J)(E_T+E'+E-E_{^8\mathrm{Be}})^5/(E_T+E_{\mathrm{r}}^J)^5$, where, for $J=0$, the photon threshold energy E_T is $7.367-4.439 = 2.928$ MeV and E_{r}^J, $\Gamma_\alpha(^{12}\mathrm{C}^J)$ and $\Gamma_\gamma(^{12}\mathrm{C}^J)$ are given in the Table. In these expressions, we follow DE87 in not including a separate non-resonant contribution as in LA86c. Indeed, such a contribution interferes with the tail of a broad resonance, and the microscopic calculation in DE87 shows that a Breit-Wigner expression provides a fair approximation of the global terms. For $J=2$, we assume the existence of a broad resonance belonging to the same rotational band as the 0_2^+ state. The capture to the ^{12}C ground state is calculated with $E_T = 7.367$ MeV and the theoretical values [DE87] $E_{\mathrm{r}}^J = 1.75$ MeV, $\Gamma_\alpha = 0.56$ MeV and $\Gamma_\gamma = 0.2$ eV. An 80% uncertainty is adopted on the γ width. The various integrals are calculated numerically. The weak influence of higher resonances is restricted to a contribution of the 3^- resonance, for which we use $\Gamma_\gamma = 2$ meV. The experimental bound and the Γ_{γ_0} value given in the Table are used as upper and lower bounds. The rate of CA88 (which is the rate of NO85 with an additional resonance term) is always smaller than the present one, in spite of the fact that our adopted α-width of the ^{8}Be ground state is smaller. Our rate is significantly larger at low temperatures because NO85 uses $\Gamma_\alpha(^{12}\mathrm{C}^J,E';E) = \Gamma_\alpha(^{12}\mathrm{C}^J)\ P_l(E')/P_l(E_{\mathrm{r}}^J-E+E_{^8\mathrm{Be}})$. Their reduced α-width then depends in an unrealistic way on the threshold energy E. At high temperatures, our rate is much larger because of the assumed 2^+ contribution.

nucleus	J^π	E_{r} (keV)	Γ_α (eV)	Γ_γ (meV)	Ref
^{8}Be	0^+	92.12	6.8 ± 1.7	–	BE68
		92.03	5.57 ± 0.25	–	WÜ92
		92.08	5.60 ± 0.25	–	adopt
^{12}C	0_2^+	287.7	8.3 ± 1.0	3.7 ± 0.5	AJ90
	3_1^-	2274	$(34 \pm 5) \times 10^3$	< 14 ($\Gamma_{\gamma_0} = 0.31 \pm 0.04$)	AJ90

T_9	low	adopt	high	exp	ratio	T_9	low	adopt	high	exp	ratio	T_9	low	adopt	high	exp	ratio
0.01	2.13	2.93	3.89	-71	11	0.1	2.05	2.38	2.70	-24	1.1	0.8	1.96	2.27	2.58	-10	1.0
0.011	4.32	5.94	7.90	-69	11	0.11	8.34	9.64	10.9	-23	1.1	0.9	2.54	2.93	3.33	-10	1.0
0.012	4.79	6.59	8.75	-67	11	0.12	1.79	2.07	2.35	-21	1.1	1	3.01	3.48	3.95	-10	1.0
0.013	3.24	4.46	5.92	-65	11	0.13	2.35	2.72	3.09	-20	1.1	1.25	3.71	4.30	4.89	-10	1.0
0.014	1.46	2.01	2.66	-63	12	0.14	2.10	2.43	2.76	-19	1.1	1.5	3.87	4.49	5.12	-10	1.0
0.015	4.65	6.40	8.50	-62	12	0.15	1.38	1.60	1.82	-18	1.1	1.75	3.72	4.37	5.02	-10	1.0
0.016	1.11	1.53	2.03	-60	12	0.16	7.11	8.22	9.34	-18	1.1	2	3.44	4.16	4.87	-10	1.1
0.018	3.07	4.22	5.61	-58	11	0.18	1.05	1.22	1.38	-16	1.1	2.5	2.86	3.92	4.99	-10	1.3
0.02	3.96	5.45	7.23	-56	8.9	0.2	0.88	1.02	1.16	-15	1.1	3	2.45	4.16	5.90	-10	1.7
0.025	0.81	1.11	1.47	-51	4.1	0.25	3.65	4.22	4.79	-14	1.1	3.5	2.22	4.77	7.39	-10	2.6
0.03	1.10	1.46	1.86	-47	2.3	0.3	3.95	4.57	5.18	-13	1.0	4	2.11	5.55	9.10	-10	3.8
0.04	4.04	5.31	6.76	-41	1.8	0.35	2.01	2.33	2.64	-12	1.0	5	2.05	7.04	12.2	-10	7.0
0.05	0.79	1.04	1.32	-36	1.7	0.4	6.48	7.49	8.50	-12	1.0	6	2.05	8.03	14.3	-10	10
0.06	0.91	1.20	1.52	-33	1.7	0.45	1.54	1.78	2.02	-11	1.0	7	2.01	8.48	15.3	-10	13
0.07	2.33	3.00	3.75	-31	1.7	0.5	2.98	3.45	3.91	-11	1.0	8	1.94	8.52	15.5	-10	15
0.08	8.18	9.68	11.2	-29	1.3	0.6	7.46	8.62	9.79	-11	1.0	9	1.84	8.28	15.1	-10	15
0.09	2.18	2.52	2.87	-26	1.2	0.7	1.34	1.55	1.75	-10	1.0	10	1.73	7.90	14.5	-10	15