

## ***Master-thesis proposals for the year 2021-2022***

The subjects proposed by the [Nuclear Physics and Quantum Physics](#) research unit (joint unit of the Sciences Faculty and of the École polytechnique de Bruxelles) are mostly theoretical in nature and usually involve mathematical and numerical modeling. The formalism used is that of quantum physics and most applications include nuclear, atomic or molecular physics.

Our research unit is involved in several networks, in which we collaborate with other nuclear-physics groups, both theoretical and experimental, in Belgium and abroad. Through these networks, there are possibilities for motivated students to realize their thesis on experimental subjects, for instance at the Interuniversity Institute for High-Energy Physics at ULB, at the IKS institute of the KU Leuven or at the SCK-CEN Mol, under the joint direction of an external and internal supervisors. For further information, please contact Jean-Marc Sparenberg.

<https://www.iihe.ac.be/ulb/ulb-sujets-de-stage-et-de-memoire>

[http://academy.sckcen.be/en/Your\\_thesis\\_internship/Bachelor\\_and\\_Master\\_thesis/Topics](http://academy.sckcen.be/en/Your_thesis_internship/Bachelor_and_Master_thesis/Topics)

<https://fys.kuleuven.be/iks/ns/phd-master-theses>

## **NUCLEAR ASTROPHYSICS**

### **1. Study of coupled-channel effects on the $^{12}\text{C}+\alpha$ system in nuclear astrophysics**

*J.-M. Sparenberg*

The  $^{12}\text{C}+\alpha$  radiative capture leading to  $^{16}\text{O}$  is of utmost importance in the nucleosynthesis helium-burning cycle of red giant stars [1]. Unfortunately, the corresponding reaction rate is too low to be directly measured experimentally at energies of astrophysical importance, hence the interest of theoretical estimates. However, such theoretical analyses are made complicated by the presence of  $^{16}\text{O}$  bound states lying just below the  $^{12}\text{C}+\alpha$  threshold, which are known to have an impact on the reaction cross section similar to resonances. Recently [2], a new parametrization of the  $^{12}\text{C}+\alpha$  elastic-scattering phase shifts led to the surprising conclusion that one of these bound states might have an imaginary asymptotic normalization constant (ANC). The origin of this effect is still unknown. The aim of this work is to test on simple phenomenological coupled-channel potential models (coupled square wells, Woods-Saxon...) whether it could be due to a coupling with other channels than the  $^{12}\text{C}+\alpha$  one, and to evaluate its impact on the capture cross section. To do so, analytical and numerical calculations will be performed (Python program with Fortran interface).

[1] A. Coc, F. Hammache and J. Kiener, [Eur. Phys. J. A \(2015\) 51](#)

[2] O. L. Ramírez Suárez and J.-M. Sparenberg, [Phys. Rev. C 96 \(2017\) 034601](#)

## MATHEMATICAL PHYSICS

### 2. Construction of phase-equivalent potentials with supersymmetric quantum mechanics

*J.-M. Sparenberg*

Supersymmetric quantum mechanics is a very efficient tool to solve the scattering inverse problem, i.e. the construction of interaction potentials from scattering data [1,2]. In particular, SUSYQM with confluent transformations allows to deal with the unicity problem, i.e. the construction of all phase-equivalent potentials sharing scattering phase shifts but with different bound spectra. A new approach to confluent transformations was proposed a few years ago [3]. The aim of this work is to explore its interest for the problem of phase-equivalent potentials, in particular for coupled channels. For that, analytic, symbolic and numerical calculations will be used. The default programming language will be Python, interfaced with Fortran to use existing subroutines and complemented by Mathematica if needed; a GPU (Graphical User Interface) for SUSYQM inversion could also be developed in the framework of the PP (Potential Program) open-source project [4]. If successful, the method will be applied to the construction of deep exactly-solvable nucleon-nucleon potentials [5], which could serve as a basic ingredient for all nuclear-physics structure and reaction calculations.

- [1] D. Baye and J.-M. Sparenberg, *J. Phys. A* 37 (2004) 10223
- [2] D. Baye, J.-M. Sparenberg, A. M. Pupasov, B. F. Samsonov, [J. Phys. A 47 \(2014\) 243001](#)
- [3] D. Bermudez et al., *Phys. Lett. A* 376 (2011) 692
- [4] <https://github.com/jmspar/PP>
- [5] J.-M. Sparenberg, [Eur. Phys. Lett. 59 \(2002\) 507](#)

### 3. Extension of the reaction-matrix method to the complex plane

*J.-M. Sparenberg and J. Dohet-Eraly*

The R-matrix (reaction-matrix) method is a powerful theoretical tool, originally introduced by Wigner and Eisenbud, to study bound and scattering states in quantum scattering theory [1]. Though usually used for real energies, it naturally leads to an analytical extension to complex energies for key quantities such as the scattering matrix, hence providing a direct access to bound states and resonant states in the complex plane.

The purpose of this work is to study this complex extension of the R-matrix, in particular to calculate the Jost function, which is an even more fundamental quantity than the scattering matrix [2]. The calculations will be performed both analytically and numerically (programming language: Python), with the use of the Lagrange-mesh technique [3]. The method will be tested on exactly-solvable models like the Bargmann [4] or the Morse [5] potentials. It will then be applied to typical systems in nuclear astrophysics (alpha+nucleon, alpha+alpha...) or in molecular physics (diatomic molecules), depending on the student's interests. Three-body applications (halo nuclei, triatomic molecules) could also be considered [3], in a second step.

- [1] P. Descouvemont and D. Baye, [Rep. Prog. Phys. 73 \(2010\) 036301](#)
- [2] S. A. Rakityansky and N. Elander, [J. Math. Phys. 54 \(2013\) 122112](#)
- [3] D. Baye, [Phys. Rep. 565 \(2015\) 1](#)
- [4] D. Baye, J.-M. Sparenberg, A. M. Pupasov, B. F. Samsonov, [J. Phys. A 47 \(2014\) 243001](#)
- [5] C. Rawitscher, [Am. J. Phys. 70 \(2002\) 935](#)